

Homework 2

Due: Friday, Oct. 2, 2009

1. Grafting, the uniting of the stem of one plant with the stem or root of another, is widely used commercially to grow the stem of one variety that produces fine fruit on the root system of another variety with a hardy root system. Most Florida sweet oranges grow on trees grafted to the root of a sour orange variety. The density for X , the number of grafts that fail in a series of five trials, is given by Table 1.
 - (a) find $P[X = 5]$
 - (b) Find the table for F (cdf)
 - (c) Use F to find the probability that at most three grafts fail; that at least two grafts fail.
 - (d) Use F to verify that the probability of exactly three failures is .03.
 - (e) Find $E[X]$, $E[X^2]$, $Var[X]$, σ_X .
 - (f) What physical unit is associated with σ_X ?

Table 1: Question 1

x	0	1	2	3	4	5
$f(x)$.7	.2	.05	.03	.01	?

2. The probability that a wildcat well will be productive is $1/13$. Assume that a group is drilling wells in various parts of the country so that the status of one well has no bearing on that of any other. Let X denote the number of wells drilled to obtain the first strike.
 - (a) Verify that X is geometric and identify the value of the parameter p .
 - (b) What is the exact expression for the density for X ?
 - (c) What are the numeric values of $E[X]$, $E[X^2]$, σ^2 , σ ?
 - (d) Find $P[X \geq 2]$.
 - (e) Suppose that 10 wells have been drilled and no strikes have occurred. What is the probability that at least 2 more wells must be drilled in order to obtain the first strike? What property can be used to answer this question?
3. Albino rats used to study the hormonal regulation of a metabolic pathway are injected with a drug that inhibits body synthesis of protein. The probability that a rat will die from the drug before the experiment is over is .2. If 10 animals are treated with the drug, how many are expected to die before the experiment ends? What is the probability that at least eight will survive? Would you be surprised if at least five died during

the course of the experiment? Explain, based on the probability of this occurring.

4. In humans, geneticists have identified two sex chromosomes, R and Y. Every individual has an R chromosome, and the presence of a Y chromosome distinguishes the individual as male. Thus the two sexes are characterized as RR (female) and RY (male). Color blindness is caused by a recessive allele on the R chromosome, which we denote by r. The Y chromosome has no bearing on color blindness. Thus relative to color blindness, there are three genotypes for females and two for males:

Table 2: Question 4

Female	Male
RR (normal)	RY (normal)
Rr (carrier)	rY (color-blind)
rr (color-blind)	

A child inherits one sex chromosome randomly from each parent.

- (a) A carrier of color blindness parents a child with a normal male. Construct a tree to present the possible genotypes for the child. Use the tree to find the probability that a given child will be a color-blind male.
- (b) If the couple has five children, what is the expected number of color-blind males? What is the probability that three or more will be color-blind males?
5. In scanning electron microscopy photography, a specimen is placed in a vacuum chamber and scanned by an electron beam. Secondary electrons emitted from the specimen are collected by a detector and an image is displayed on a cathode-ray tube. This image is photographed. In the past a 4- × 5-inch camera has been used. It is thought that a 35-millimeter camera can obtain the same clarity. This type of camera is faster and more economical than the 4- × 5-inch variety.
- (a) Photographs of 15 specimens are made using each camera system. These unmarked photographs are judged for clarity by an impartial judge. The judge is asked to select the better of the two photographs from each pair. Let X denote the number selected taken by a 35-mm camera. If there is really no difference in clarity and the judge is randomly selecting photographs, what is the expected value of X ?
- (b) Would you be surprised if the judge selected 12 or more photographs taken by the 35-mm camera? Explain, based on the probability involved.
- (c) If $X \geq 12$, do you think that there is reason to suspect that the judge is not selecting the photographs at random?

6. Let X be a Poisson random variable with parameter $k = 10$.
- (a) Find $E[X]$
 - (b) Find $Var[X]$
 - (c) Find σ_X .
 - (d) Find $P[X \leq 4]$
 - (e) Find $P[X < 4]$
 - (f) Find $P[X = 4]$
 - (g) Find $P[X \geq 4]$
 - (h) Find $P[4 \leq X \leq 9]$.
7. In *Escherichia coli*, a bacterium often found in the human digestive tract, 1 cell in every 10^9 will mutate from streptomycin sensitivity to streptomycin resistance. This mutation can cause the individual involved to become resistant to the antibiotic streptomycin. In observing 2 billion (2×10^9) such cells, what is the probability that none will mutate? What is the probability that at least one will mutate?