

Homework 1

Due: Friday, Sept. 18, 2009

1. When an individual is exposed to radiation, death may ensue. Factors affecting the outcome are the size of the dose, the length and intensity of the exposure, and the biological makeup of the individual. The term LD_{50} is used to denote the dose that is usually lethal for 50% of the individuals exposed to it. Assume that in a nuclear accident 30% of the workers are exposed to the LD_{50} and die; 40% of the workers die; and 68% are exposed to the LD_{50} or die. What is the probability that a randomly selected worker is exposed to the LD_{50} ? Use a Venn diagram to find the probability that a randomly selected worker is exposed to the LD_{50} but does not die. Find the probability that a randomly selected worker is not exposed to the LD_{50} but dies.
2. A test has been developed to detect a particular type of arthritis in individuals over 50 years old. From a national survey it is known that approximately 10% of the individuals in this age group suffer from this form of arthritis. The proposed test was given to individuals with confirmed arthritis disease, and a correct test result was obtained in 85% of the cases. When the test was administered to individuals of the same age group who were known to be free of the disease, 4% were reported to have the disease. What is the probability that an individual has this disease given that the test indicates its presence?
3. Prove the De Morgan's law and illustrate by using a Venn diagram:

$$(A \cup B)^c = A^c \cap B^c, \quad (A \cap B)^c = A^c \cup B^c$$

4. The ability to observe and recall details is important in science. Unfortunately, the power of suggestion can distort memory. A study of recall is conducted as follows: Subjects are shown a film in which a car is moving along a country road. There is no barn in the film. The subjects are then asked a series of questions concerning the film. Half the subjects are asked, "How fast was the car moving when it passed the barn?" The other half is not asked the question. Later each subject is asked: "Is there a barn in the film?" Of those asked the first question concerning the barn, 17% answer "yes"; only 3% of the others answer "yes." What is the probability that a randomly selected participant in this study claims to have seen the nonexistent barn? Is claiming to see the barn independent of being asked the first question about the barn?
5. A box contains 6 red balls, 4 white balls, and 5 blue balls. Three balls are drawn successively from the box. Find the probability that they are drawn in the order red, white, and blue if each ball is (a) replaced, (b) not replaced.

6. The probability that a unit of blood was donated by a paid donor is .67. If the donor was paid, the probability of contracting serum hepatitis from the unit is .0144. If the donor was not paid, this probability is .0012. A patient receives a unit of blood. What is the probability of the patient's contracting serum hepatitis from this source?
7. Theorem: If an event A must result in one of the mutually exclusive events A_1, A_2, \dots, A_n , then

$$P(A) = P(A_1)P(A|A_1) + P(A_2)P(A|A_2) + \dots + P(A_n)P(A|A_n)$$

Prove this theorem.

8. Studies in population genetics indicate that 39% of the available genes for determining the Rh blood factor are negative. Rh negative blood occurs if and only if the individual has two negative genes. One gene is inherited independently from each parent. What is the probability that a randomly selected individual will have Rh negative blood?
9. In how many ways can 7 people be seated at a round table if (a) they can sit anywhere, (b) 2 particular people must not sit next to each other?
10. A power network involves three substations A, B and C . Overloads at any of these substations might result in a blackout of the entire network. Past history has shown that if substation A alone experiences an overload, then there is a 1% chance of a network blackout. For stations B and C alone these percentages are 2% and 3%, respectively. Overloads at two or more substations simultaneously result in a blackout 5% of the time. During a heat wave there is a 60% chance that substation A alone will experience an overload. For stations B and C these percentages are 20% and 15%, respectively. There is a 5% chance of an overload at two or more substations simultaneously. During a particular heat wave, a blackout due to an overload occurred. Find that probability that the overload occurred at substation A alone; substation B alone; substation C alone; two or more substations simultaneously.
11. A person places n different letters into n differently addressed envelopes at random. Find the probability that at least one of the letters will arrive at the proper destination.
12. How many 4-digit numbers can be formed with the 10 digits $0, 1, 2, 3, \dots, 9$ if (a) repetitions are allowed, (b) repetitions are not allowed, (c) the last digit must be zero and repetitions are not allowed?